Section 11.11 Solutions

math 31

4. The ~~Taylo~~ Maclaurin Series $(a=0)$ for e^{-x} can be obtained from the Maclaurin series for e^x by substitution

$$e^{-x} = \sum_{n=0}^{\infty} \frac{(-x)^n}{n!} = \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n!} = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \cdots$$

So T_3 for $f(x)$ about 0 is $x + e^{-x} \approx x + \left[1 - x + \frac{x^2}{2} - \frac{x^3}{6}\right]$

$$\boxed{T_3 = 1 + \frac{x^2}{2} - \frac{x^3}{6}}$$

6. $f(x) = e^{-x}\sin x$ $\qquad f(0) = 0$

$f'(x) = -e^{-x}\sin x + e^{-x}\cos x$ $\qquad f'(0) = 1$

$f''(x) = (e^{-x}\sin x - e^{-x}\cos x) + (-e^{-x}\cos x - e^{-x}\sin x)$

$= -2e^{-x}\cos x$ $\qquad f''(0) = -2$

$f'''(x) = -2[-e^{-x}\cos x - e^{-x}\sin x]$

$= 2e^{-x}(\cos x + \sin x)$ $\qquad f'''(0) = 2$

$$T_3 = x - \frac{2x^2}{2!} + \frac{2x^3}{3!} = \boxed{x - x^2 + \frac{x^3}{3}}$$

8. $f(x) = x\cos x$ $\qquad f(0) = 0$

$f'(x) = \cos x - x\sin x$ $\qquad f'(0) = 1$

$f''(x) = -\sin x - \sin x - x\cos x$

$= -2\sin x - x\cos x$ $\qquad f''(0) = 0$

$f'''(x) = -2\cos x - \cos x + x\sin x$

$= -3\cos x + x\sin x$ $\qquad f'''(0) = -3$

$$T_3 = x - \frac{3x^3}{3!} = \boxed{x - \frac{x^3}{2}}$$

OR

Note that $x\cos x = x\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n)!}$

$= x - \frac{x^3}{2!} + \frac{x^5}{4!} - \frac{x^7}{6!}$, and

$$\boxed{T_3 = x - \frac{x^3}{2!}}$$

10. $f(x) = \tan^{-1} x$ $\qquad f(1) = \pi/4$

$f'(x) = \frac{1}{1+x^2} = (1+x^2)^{-1}$ $\qquad f'(1) = 1/2$

$f''(x) = -(1+x^2)^{-2}(2x)$ $\qquad f''(1) = -1/2$

$= -2x(1+x^2)^{-2}$

$f'''(x) = -2(1+x^2)^{-2} + 8x^2(1+x^2)^{-3}$ $\qquad f'''(1) = 1/2$

so $\boxed{T_3 = \frac{\pi}{4} + \frac{1}{2}(x-1) - \frac{1}{4}(x-1)^2 + \frac{1}{12}(x-1)^3}$